CONFINEMENT OF THE SUN'S INTERIOR MAGNETIC FIELD, WITH IMPLICATIONS FOR LITHIUM BURNING

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ABSTRACT

The simplest interior magnetic field $B_i$ that can explain the observed uniform rotation of the Sun’s radiative envelope is an axial dipole stabilized by a deep toroidal field. It can explain the uniform rotation only if confined in the polar caps. The field must be prevented from diffusing up into the high-latitude convection zone, whose slower rotation must remain decoupled from the radiative interior. This paper describes new analytical and numerical solutions of the relevant magnetohydrodynamic equations showing that such confinement and decoupling is dynamically possible by means of a laminar “magnetic confinement layer” at the bottom of the tachocline. With realistic values of the microscopic diffusivities, a weak laminar downwelling flow $U \sim 10^{-5}$ cm s$^{-1}$ over the poles is enough to enforce exponential decay of $|B_i|$ with altitude, in a confinement layer only a fraction of a megameter thick. Downwelling in the polar tachocline is implied both by helioseismic observations, combined with elementary dynamics, and by theoretical arguments about the “gyroscopic pumping” that would spread differential rotation downward in the absence of $B_i$. Our confinement-layer solutions are the first to take account of all the relevant physical effects in a self-consistent mathematical model. The effects include magnetic diffusion, baroclinicity and stable stratification (thermal and compositional), Coriolis effects, and thermal relaxation. We discuss how the confinement layers at each pole might fit into a global dynamical picture of the solar tachocline. That picture, in turn, suggests a new insight into the early Sun and solar lithium depletion.

Subject headings: MHD — Sun: abundances — Sun: interior — Sun: magnetic fields — Sun: rotation

1. INTRODUCTION

There is increasing evidence that a purely hydrodynamic theory of the solar tachocline will not work (e.g., Gough & McIntyre 1998, hereafter GM98). It seems clear that a global-scale interior magnetic field $B_i$ must be involved as well as Coriolis effects, stable stratification, baroclinicity, and thermal relaxation. In order to describe a realistically thin tachocline under dynamical conditions resembling those in today’s Sun, it is crucial to take account of angular momentum transport by global-scale Maxwell stresses as well as by mean meridional circulations (MMCs).

The first attempt at a tachocline theory was that of Spiegel & Zahn (1992). It included all of the above effects except $B_i$. Rüdiger & Kitchatinov (1997) included $B_i$ but omitted the other effects. The first attempt to include all of them was that of GM98, in a line of investigation further developed by Garaud & Garaud (2008). Meanwhile, the dynamical importance of compositional as well as thermal stratification (e.g., Mestel 1953) was suggested for tachocline theories (McIntyre 2007). In particular, the helium settling layer beneath the tachocline is nearly impermeable to MMCs because of the small diffusivity of helium through hydrogen. This near-impermeability of compositionally stratified regions has been called the “nu-choke” (Mestel & Moss 1986). The reality of the Sun’s helium settling layer is strongly indicated both by standard solar-evolution models and by helioseismology (e.g., Christensen-Dalsgaard et al. 1993; Ciaccio et al. 1997; Elliott & Gough 1999; Christensen-Dalsgaard & Thompson 2007).1

1 Also Christensen-Dalsgaard & Gough 2010, in preparation.
The need for the interior field \( \mathbf{B}_i \) arises from a well-known difficulty with purely hydrodynamic theories. They tend to spread the strong differential rotation of the convection zone down into the radiative interior. This is a robust and well-understood consequence of thermal relaxation, interacting with Coriolis effects and gyroscopically-pumped MMCs (Haynes et al. 1991; Spiegel & Zahn 1992; Elliott 1997; McIntyre 2007; Garaud & Brummell 2008). As pointed out by Spiegel and Zahn, this downward spreading or burrowing would have produced a tachocline far thicker than observed. It would also have prevented the helium settling layer from forming.

To counter the burrowing tendency and allow the interior to rotate uniformly, angular momentum has to be transported somehow from the low-latitude tachocline to the high-latitude tachocline. The horizontal eddy viscosity proposed for this purpose by Spiegel & Zahn is inconsistent with the properties of stratified turbulence known from many studies of the terrestrial atmosphere and oceans (e.g., McIntyre 1994, 2003, & refs.). Radial transport by internal gravity waves is a physically possible alternative (e.g., Schatzman 1993; Zahn et al. 1997; Rogers & Glatzmaier 2006; Charbonnel & Talon 2007, & refs.). However, it is highly improbable as the main mechanism because it has no natural tendency to produce the observed uniform interior rotation.

A suitably-shaped magnetic field can, by contrast, naturally produce the required angular momentum transport, as is well known. A suitable shape is one in which field lines link low latitudes to high latitudes. The simplest such shape is that suggested in Fig. 1, in which the linkage is via a time-averaged field whose lines thread the tachocline, forming the superficial part of a global-scale interior dipole stabilized by a deep toroidal field (e.g., Braithwaite & Spruit 2004). Such an interior dipole imposes a “Ferraro constraint” on the interior that helps to enforce uniform rotation. It is crucial that the field lines emerging from the interior (light-gray sphere) bend over toward the horizontal as they enter the tachocline. We call such a field “confined”, essentially meaning “confined beneath the convection zone”. In particular, the field lines must be prevented from extending upward through the polar cap, as is often seen in numerical simulations in which magnetic diffusion dominates (e.g., Braithwaite & Spruit 2004; Brun & Zahn 2006).

The time averaging envisaged in Fig. 1 conceals a plethora of fast processes, including the 22-year dynamo cycle, convective overshoot, and other turbulent processes. We presume that the effect of the fast processes is qualitatively like that of a turbulent magnetic diffusivity that stops the field being wound up arbitrarily tightly by the shear in the tachocline — hence the inflected shapes of the time-averaged field lines shown in Fig. 1.

![Fig. 1.— Sketch of the time-averaged magnetic field in the tachocline, taken from Wood & McIntyre (2010). Poloidal magnetic field lines emerge from the helium settling layer (inner sphere) in high latitudes, and are wound up by differential rotation in the tachocline, acting against eddy diffusion. A prograde torque is transmitted from low to high latitudes along these field lines. The slow polar and fast equatorial rotation are indicated by darker shading of the outer sphere, which represents the bottom of the convection zone. The dashed lines indicate the latitudes at which the rotation of the convection zone matches that of the interior, \( \Omega_i = 2.7 \times 10^{-6} \text{s}^{-1} \).

We presume furthermore that away from the poles the field lines are held down, and held approximately horizontal, by turbulent “magnetic flux pumping” in the overshoot layer. Though not fully understood, the effectiveness of such flux pumping in the overshoot layer can be strongly argued from several lines of evidence, including three-dimensional direct numerical simulations, with varying emphasis on the role of turbulent
anisotropy and of vertical gradients of density and turbulent intensity (e.g., Tobias et al. 2001; Kitchatinov & Rüdiger 2008, & refs.) (see also §3 of Weiss et al. 2004, for a historical review). The shapes of the time-averaged $B$ lines in Fig. 1 are evidently such as to transport angular momentum from low to high latitudes by means of persistent Alfvénic torques.

Near the poles it is less clear that magnetic flux pumping will be effective in confining the field. However, as argued for instance in GM98, there are good reasons (§2 below) to expect the tachocline’s MMC near the poles to take the form of weak but persistent downwelling. This suggests that the field can be confined in the polar caps through a laminar advective–diffusive balance, the kind of balance proposed in GM98. That such confinement is possible — taking account of all the abovementioned dynamical ingredients, including strong Coriolis effects — was first shown in detail by the present authors (Wood & McIntyre 2007, hereafter WM07) by solving the relevant nonlinear equations in the limits of infinitely strong stable stratification and infinitely weak viscosity and helium diffusivity. The steady, magnetically diffusive, inviscid flow configuration thus described was called a “magnetic confinement layer”.

Apart from the fully consistent dynamics, a distinctive feature of these confinement-layer solutions — contrasting with published numerical solutions that seemed to argue against polar confinement (e.g., Brun & Zahn 2006) — is the ability to use realistically small values of the microscopic magnetic and thermal diffusivities. However, in WM07 we assumed that the angular momentum balance in the polar cap would include a retrograde Alfvénic torque exerted downward on the helium settling layer beneath. We now recognize this to be inconsistent with the global picture suggested in Fig. 1, though it could be made consistent, topologically at least, with the next simplest picture involving a quadrupolar interior field. Here we focus on the simplest, dipolar picture.

In this and a companion paper (Wood & McIntyre 2010, hereafter WM10) we present polar confinement-layer solutions consistent with Fig. 1, that is, with all the Alfvénic torque exerted sideways. We also generalize the solutions to allow finite thermal and compositional stratifications and finite viscosity and helium diffusivity. The flows are again hypothesized to be laminar, and microscopic diffusivity values are assumed throughout.

With finite helium diffusivity a very thin “helium sublayer” forms at the bottom of the confinement layer, just above the helium settling layer. The sublayer replaces the Ekman layer whose existence was conjectured in WM07. No Ekman layer forms because, with realistic parameters, helium diffusion has much more time to act against advection than viscosity against Coriolis effects. This limits the magnitude of the shear such that the flow remains effectively inviscid.

The Coriolis torques within the confinement layer are balanced by Lorentz torques. In this way the magnetic field prevents the confinement layer’s MMC from burrowing into the interior and thickening the tachocline.

We also extend our confinement-layer model to include cases with no helium settling layer, and hence no mu-choke, as in the early Sun. We show that the magnetic field can still be confined and the burrowing tendency stopped. In such cases the helium sublayer is replaced by a thicker structure, but the flow remains laminar and inviscid. That result has relevance to the Sun’s early main-sequence evolution. It explains for instance how the burrowing tendency could have been held at bay from the start, allowing the helium settling layer to form. It will also suggest a new insight into how the Sun’s surface lithium was depleted.

The present paper is concerned with the physics of the polar confinement-layer solutions, and their astrophysical significance, while the companion paper WM10 is concerned with the equations and the detailed mathematical techniques, analytical and numerical, used to solve the equations and cross-check the results. WM10 also makes a prima facie case for dynamical stability, following WM07, consistent with our hypothesis that the polar confinement-layer flows are laminar.

The plan of the present paper is as follows. In §2 we summarize the reasons for expecting persistent downwelling over the poles. In §§3 and 4 we describe in detail a typical confinement-layer solution relevant to today’s Sun, with emphasis on the physical processes involved including the angular momentum balance and the roles of the magnetic and compositional diffusivities. In §5 we present a confinement-layer solution relevant to the early
Sun, with no helium settling layer and no helium sublayer. In the concluding discussion, §6, we consider the implications for early solar evolution and lithium depletion.

2. DOWNWELLING IN THE POLAR TACHOCLINE

Our polar-confinement scenario relies on the MMC pattern in the stably-stratified polar tachocline being robustly and persistently downward above the confinement layer, after averaging out any fast fluctuations due to waves and turbulence. Helioseismology provides a compelling reason to expect downwelling rather than upwelling, at least in today’s tachocline. A further reason is that a downward MMC over the pole is a robust consequence of the gyroscopic pumping already mentioned, which, in the absence of the interior field \( B_i \), would mediate the downward spreading or burrowing of the convection zone’s slow polar rotation. The distinction between gyroscopically-pumped MMCs and MMCs driven in other ways is reviewed in McIntyre (2007, §§8.1–8.2). Thus we expect polar downwelling to be robust not only in today’s Sun, but also throughout the Sun’s main-sequence lifetime.

The argument from helioseismology is as follows. As is well known, the pressure, density and angular velocity fields, averaged with respect to time and longitude, satisfy hydrostatic and cyclonic balance to excellent approximation. Departures from such balance must take the form of fast oscillations such as \( p \)-modes and \( g \)-modes, or turbulent fluctuations. From the curl of the momentum equation, taking its azimuthal component, we may show in the standard way that balance implies the so-called “thermal-wind relation”. In cylindrical polar coordinates \((\rho, \zeta, \phi)\) centered on the axis of rotation, with the axial coordinate \(z\) directed vertically upward at the north pole, the thermal-wind relation can be expressed as

\[
\left( \nabla \rho \times \nabla \rho \right) \cdot \mathbf{e}_\phi = -\rho \partial z \frac{\partial |\boldsymbol{\Omega}|^2}{\partial z} \tag{1}
\]

where \( \boldsymbol{\Omega} \) is the absolute angular velocity of the Sun’s differential rotation, \( \rho \) is the density, and \( p \) is the total pressure. The unit vector \( \mathbf{e}_\phi \) is directed azimuthally, while \( \nabla \rho \), being dominated by its hydrostatic part, is very close to being vertically downward. On the assumption that the observed negative sign of \( \partial |\boldsymbol{\Omega}|^2/\partial z \) persists into the region near the pole invisible to helioseismology, we must have a minimum in \( \rho \), and hence a maximum in temperature \( T \), on each isobaric surface at the pole.

The stably-stratified radiative envelope is a thermally relaxing system. Local temperature anomalies, defined as departures of \( T \) from local radiative equilibrium, will tend to relax back toward zero. To hold \( T \) above radiative equilibrium near the pole, there has to be persistent adiabatic compression by downwelling, with compensating upwelling and negative \( T \) anomalies in lower latitudes.

The strength \( U \) of the polar downwelling is difficult to estimate precisely. Among other things it depends on the tachocline thickness, which is not well constrained by helioseismology. The thickness scale enters both via equation (1) and, more sensitively, via the rate of diffusive thermal relaxation within the tachocline. GM98 estimated \( U \sim 10^{-5}\text{cm s}^{-1} \), using the rather small thickness estimate, 13 Mm, derived by Elliott & Gough (1999). The value of \( U \) estimated in this fashion is inversely proportional to the cube of the tachocline thickness, and so a similar estimate using a deeper tachocline would yield a much smaller value of \( U \).

However, GM98 assumed that the bulk of the tachocline is laminar. McIntyre (2007) considered an alternative scenario in which magnetohydrodynamic turbulent stresses within a deeper tachocline dominate the angular-momentum transport from the overlying convection zone, except near the bottom of the tachocline. The turbulent stresses were estimated by assuming a particular prescription for the turbulence, following Spruit (2002). The stresses diverge in a thin layer near the bottom of the tachocline, just above the confinement layer, where they gyroscopically pump a downwelling of the order of \( U \sim 4 \times 10^{-6}\text{cm s}^{-1} \) or greater.

Fortunately, our confinement-layer solutions can accommodate a wide range of uncertainty over the value of \( U \). They will show that polar field confinement by downwelling is robust over a range of \( U \) values at least as wide as \( 10^{-6} \text{cm s}^{-1} \) to \( 10^{-4} \text{cm s}^{-1} \). From here on we use GM98’s value \( U \sim 10^{-5}\text{cm s}^{-1} \) for illustrative purposes.
Fig. 2.— Streamlines and magnetic field lines in a confinement layer appropriate to today's Sun. The field strength falls off exponentially with altitude $z$. If the downwelling were switched off, then the field near the pole would diffuse and become nearly vertical throughout the domain. Compositional stratification is indicated by shading. From WM10; q.v. for further details.

3. THE CONFINEMENT LAYER IN TODAY'S SUN

Figure 2 is a vertical cross-section through a typical confinement-layer solution appropriate to today's Sun, in a region surrounding the north pole. The streamlines with arrows show the MMC responsible for confining the magnetic field $B$. The shading indicates the compositional stratification at the top of the helium settling layer. Figure 3 gives perspective views of the same solution. The top and bottom panels show some of the three-dimensional streamlines and magnetic field lines. Those omitted can be visualized by uniformly shrinking or expanding the horizontal scale, as suggested by comparing the views in Fig. 3 with that in Fig. 2. The horizontal structure is approximately self-similar.

The field $B$ is well confined in the sense that all three components of $B$ die off exponentially with altitude. The axes in the figures are marked in units of the magnetic advection–diffusion e-folding scale $\delta = \eta/U$, where $\eta$ is the magnetic diffusivity. Taking $U \sim 10^{-5}\text{cm s}^{-1}$ and $\eta \approx 4.1 \times 10^2\text{cm}^2\text{s}^{-1}$ (Gough 2007), we find $\delta \sim 0.4\text{Mm}$, which may be compared with 13Mm, the smallest published tachocline thickness estimate (Elliott & Gough 1999).

The Alfvénic torque transmitted by the curved field lines is an essential part of how the flow is held in a steady state and the burrowing tendency stopped. One way to see this is to consider the angular momentum balance integrated over the cylindrical region shown in Fig. 3. The MMC by itself would start to spin the region down — the initial stage of the burrowing process — because the rate at which prograde angular momentum is exported through the sides of any such cylinder exceeds the rate at which it is imported through the top. However, this effect of the MMC is offset by the prograde Alfvénic torque over the side of the cylinder, anticipated from Fig. 1. There is no torque on the bottom.

More precisely, in the steady state we may write the angular-momentum equation integrated over the cylinder, denoted $V$, say, as

$$
\int_{\partial V} r \rho u_\phi \mathbf{u} \cdot d\mathbf{S} = \frac{1}{4\pi} \int_{\partial V} r B_\phi \mathbf{B} \cdot d\mathbf{S} \quad (2)
$$

where $r$ is colatitudinal distance and $\rho$ density, as in (1), while $u_\phi \approx \Omega r > 0$ is the prograde azimuthal velocity in an inertial reference frame, $\partial V$ is the boundary of $V$, and $d\mathbf{S}$ is the area element.
directed outward, across which there is a mass flux \( \rho \mathbf{u} \cdot \mathbf{dS} \) due to the meridional part of the velocity field \( \mathbf{u} \). The left-hand side of (2) is positive for any velocity field qualitatively like that in Fig. 2. The right-hand side, representing the Alfvénic torque across the side of the cylinder, must therefore be positive as well, with \( B_\phi \) the azimuthal component of the magnetic field \( \mathbf{B} \) in Gaussian cgs units. Viscous torques have been neglected. That neglect is justified, as shown in WM10, by the abovementioned fact that no Ekman layer forms. This is related to the fact that the Ekman-layer thickness scale is several orders of magnitude smaller than the thickness of the helium sublayer.

The equations solved are the standard Boussinesq and magnetic induction equations for a thin layer, written in a frame of reference rotating with angular velocity \( \Omega \). As is well known, the Boussinesq equations, including the simplified mass-conservation equation \( \nabla \cdot \mathbf{u} = 0 \), are valid in the limit of small height-scale ratio \( \delta/H_p \) and small Mach number \( U/c_s \), where \( H_p \approx 60 \text{ Mm} \) is the pressure scale height and \( c_s \approx 2.3 \times 10^7 \text{ cm s}^{-1} \) is the sound speed. With \( U \sim 10^{-5} \text{ cm s}^{-1} \) we have \( \delta/H_p \sim 10^{-2} \) and \( U/c_s \sim 10^{-12} \). The equations are nonlinear, but have proved amenable to solution using two different techniques that cross-check each other. The first is purely numerical, using finite differences, and the second is analytical in the sense that the fields are obtained explicitly, as indefinite integrals, after first solving a linear second order ordinary differential equation for the vertical component of \( \mathbf{B} \). Full details are given in WM10.

The analytical solutions are valid in the limit of infinitely strong stable stratification. As shown in WM10 and further discussed in the next section, the thermal and compositional stratification surfaces then become perfectly flat and the horizontal structure exactly self-similar. Perfectly flat means exactly horizontal, corresponding in the real Sun to coincidence with the effective gravitational heliopotentials with allowance for the centrifugal contribution.

By contrast, the purely numerical solutions are valid for finite stratifications. We use them to demonstrate directly that the assumption of infinite stratification and perfect flatness is a good approximation for realistic solar parameters. The solution shown in Figs. 2 and 3 is a numerical solution. It is barely distinguishable from its analytical counterpart. In particular, the departure from perfect flatness in the helium sublayer, at the top of the shaded region in Fig. 2, is barely visible even when one looks at the figure edge-on. The thermal stratification surfaces, not shown, are also found to be almost perfectly flat.

We note briefly how the flow is described by the equations in the rotating frame. Because the flow relative to that frame is so slow, by comparison with the absolute velocity \( \Omega \mathbf{e}_\phi \) of the frame itself, Coriolis accelerations dominate relative accelerations by many orders of magnitude. At any point in the equatorward flow, therefore, there is a retrograde Coriolis force almost perfectly in balance with a prograde Lorentz force from the curved field lines.

The field is shaped to provide that Lorentz force by a subtle interplay between magnetic diffusion, advection, stretching, and twisting. In particular, the twisting of field lines by the differential rotation of the relative \( u_\phi \) field seen in Fig. 3a has a crucial role, along with advection and stretching by the MMC. WM10 gives the full mathematical details.

4. VERTICAL AND HORIZONTAL SCALES

Because of the small value of the diffusivity \( \chi \) of helium through hydrogen, \( \chi \approx 0.9 \times 10^1 \text{ cm}^2\text{s}^{-1} \) (Gough 2007), the helium settling layer is very nearly impermeable to the flow shown in Figs. 2 and 3a. This is the “mu-choke” effect described by Mestel & Moss (1986). It means that the vertical velocity component \( u_z \) approaches zero as the flow encounters the top of the helium settling layer. The thickness scale \( \delta \chi \) of the helium sublayer is therefore governed by the vertical strain-rate \( \partial u_z/\partial z \sim U/\delta \) rather than by \( u_z \) itself. This strain rate must match the rate of helium diffusion across the sublayer, \( \chi/\delta^2 \). Hence

\[
\chi/\delta^2 = U/\delta = U^2/\eta ,
\]

implying that

\[
\delta = (\chi/\eta)^{1/2}/U .
\]

So \( \delta/\delta = (\chi/\eta)^{1/2} \approx 0.15 \). Owing to the thinness of the sublayer in comparison with the confinement layer, the flow within the sublayer induces only slight distortions of the magnetic field lines, as can be seen in Fig. 2. One consequence, discussed in WM10, is that the flow through the
sublayer has a dynamics similar to flow through a porous medium, subject to so-called Darcy friction. To that extent it is like Ekman-layer dynamics, because Coriolis accelerations dominate relative accelerations, as they do throughout the confinement layer. However, the Darcy friction is not to be confused with the real viscous friction that might enter the problem if the viscosity were much greater than it is in reality. The negligible role of the real viscosity has already been mentioned and is further discussed in WM10.

Figures 2 and 3 show only a small neighborhood of the pole. However, the self-similarity of the horizontal flow structure suggests that the confinement layer might be able to cover a larger area. There are two separate constraints on how large that area can be. The first is the area over which the gyroscopic pumping from above can supply an approximately uniform downwelling. This is hard to estimate without detailed theories of convection-zone turbulence and of magnetohydrodynamic turbulence within the tachocline, and will be left aside for future modeling studies.

The second constraint is that, even under uniform downwelling, the confinement-layer dynamics changes its character, and may even break down altogether, at a certain colatitudinal distance \( r \sim r_T \delta \), say, at which the stable stratification fails to hold the stratification surfaces sufficiently flat to permit solutions qualitatively like those described here. On the basis of a careful scale analysis WM10 concludes that, for realistic solar parameter values, it is the thermal rather than the compositional stratification that determines \( r_T \) in today’s Sun. The strength of the thermal stratification can be quantified in terms of the thermal buoyancy frequency \( N_T \), which for an ideal gas is defined by

\[
N_T^2 = \left( g / H_p \right) \left( \nabla_{ad} - \nabla \right)
\]

where \( g \) is the local gravitational acceleration, \( H_p \) is again the pressure scale height, and \( \nabla_{ad} \) and \( \nabla \) are the adiabatic and actual logarithmic vertical derivatives of temperature with respect to pressure. Near the bottom of the tachocline this gives \( N_T \approx 0.8 \times 10^{-3} \text{s}^{-1} \) (Gough 2007). In terms of \( N_T^2 \), WM10 show that \( r_T \) may be defined by

\[
r_T^2 = \frac{N_T^2 \delta^2}{2 \Omega \kappa} \min(\Lambda, \Lambda^{-1}) ,
\]

where \( \kappa \approx 1.4 \times 10^7 \text{cm}^2\text{s}^{-1} \) is the thermal diffusivity (Gough 2007) and where \( \Lambda \) is an Elsasser number defined by

\[
\Lambda = \frac{B^2}{8 \pi \rho \Omega \eta}
\]

in which \( B \) is the magnitude of the vertical component of \( B \) just beneath the confinement layer, at \( z = 0 \) in Fig. 2. The solution shown in Figs. 2 and 3 has \( \Lambda \approx 3.5 \). Then, with \( \rho \approx 2.1 \text{ g cm}^{-3} \) and with our standard magnitudes \( U \sim 10^{-5} \text{cm s}^{-1} \) and \( \delta \sim 0.4 \text{ Mm} \), we have \( r_T \delta \sim 2 \times 10^3 \delta \sim R_\odot \sim 60^\circ \) colatitude. This large value means that departures from flatness are unlikely to become dynamically important even at the largest colatitudinal distances that might be relevant. Rather, in this case, it is the colatitudinal extent of the downwelling that will govern the area covered by the confinement layer.

With \( \Lambda \sim 3.5 \) and the other numbers given above, the strength of the interior field \( B_i \) just beneath the confinement layer at, say, colatitude \( z \approx 30^\circ \) has the order of magnitude \( |B_i| \sim 10^8 \text{G} \). This uses solenoidality, \( \nabla \cdot B = 0 \), and the implied scaling relation between the vertical (\( B \)) and horizontal (\( B_r \), say) components of \( B \) in and just beneath the confinement layer, \( |B_i| \sim B_r \sim r B / \delta \). The magnitude \( |B_i| \sim 10^2 \text{G} \) is substantially greater than the threshold, more like \( 10^{-2} \text{G} \), for the field strength required to enforce the Ferraro constraint in the interior (e.g. Mestel & Weiss 1987; Charbonneau & MacGregor 1993). WM10 shows that, for \( U \sim 10^{-5} \text{cm s}^{-1} \), confinement layers of colatitudinal extent \( \gtrsim 10^\circ \) are dynamically possible for a wide range of \( \Lambda \) values, \( 10^{-2} \lesssim \Lambda \lesssim 10^2 \), corresponding to \( 10^6 \lesssim |B_i| \lesssim 10^9 \text{G} \) at the nominal \( 30^\circ \) colatitude. For smaller values of \( U \), and correspondingly larger \( \delta = \eta / U \), a still wider range of \( \Lambda \) values, hence \( |B_i| \) values, may become possible.

WM10’s detailed solutions and scale analysis show that, in the confinement layer, the ratios of the horizontal components of \( B \) and \( u \), say \( B_r / B_\phi \) and \( u_r / u_\phi \), both scale like \( \Lambda \). For larger \( \Lambda \) the field lines are stiffer and therefore straighter. For smaller \( \Lambda \) they are more curved.

5. THE CONFINEMENT LAYER IN THE EARLY SUN

Early in its main-sequence phase the Sun would have had no helium settling layer. This raises the question of whether confinement-layer dynamics
can then still work. That is, are steady confinement-layer flows possible even when there is no mu-choke to help stop the burrowing tendency? The solutions derived in WM10 show that the answer is yes. Such confinement-layer flows are indeed possible. Figure 4 shows one such solution. Since there is no helium settling layer, the $z$-origin is now arbitrary.

\[\begin{array}{c}
\Omega \\
\mathbf{B} \\
U
\end{array}\]

Fig. 4.— Streamlines and magnetic field lines in a confinement layer appropriate to the early Sun. From WM10; q.v. for further details.

In the close-up view provided by Fig. 4, the deeper penetration of the flow is conspicuous. In this example the penetration $e$-folding scale is $\delta$, about seven times greater than $\delta_\chi$. Nevertheless, the flow is steady, and therefore not burrowing. The velocity field $\mathbf{u}$ decays exponentially downward and the magnetic field $\mathbf{B}$ decays exponentially upward, also on a scale $\sim \delta$. So this is still an effective confinement layer. The interior field $\mathbf{B}_i$ is enough by itself to hold the burrowing in check, as originally proposed in GM98.

In the case of Fig. 2, by contrast, the upward decay of $\mathbf{B}$ is on the same scale $\sim \delta$ but the downward decay of $\mathbf{u}$ is on a much smaller scale, even smaller than $\delta_\chi$. A more detailed analysis (WM10, Wood 2010) shows that scales for downward decay depend on horizontal scales in a manner reminiscent of the heuristic boundary-layer theory of GM98. However, the dependence on horizontal scales is not precisely the same, even in helium-free cases, because the vertical component of $\mathbf{B}$, neglected in GM98, is always important in the polar regions.

6. CONCLUSIONS AND FUTURE DIRECTIONS

We cannot yet claim to have a complete tachocline theory, because the confinement layer and helium sublayer form only two pieces of a complicated jigsaw puzzle. Other aspects of that jigsaw include the way in which the confinement layer matches upward to the negative shear in the bulk of the tachocline, and the way in which the baroclinic temperature anomalies induced by the tachocline’s MMC ($\S 2$ above) fit into the perturbed global-scale heat flow. In particular, without putting the whole jigsaw together we cannot quantitatively predict the thickness of the tachocline. Nor can we predict the precise shapes of the vertical profiles of $\mathbf{B}$ and $\mathbf{u}$, which depend on the confinement layer’s surroundings. This is discussed in $\S 6$ of WM10.

All these aspects remain a challenge for the future. However, we have obtained the first fully consistent theory of polar field confinement and how it could work in today’s Sun.

We have also shown, in $\S 5$, that confinement can work in the early Sun. The dynamics is similar apart from the slightly deeper penetration of the MMC arising from the absence of the helium settling layer and helium sublayer. We can use the resulting insights, alongside our well-established understanding of the gyroscopic pumping of MMCs, to say something new about the early Sun and the solar lithium-burning problem.

As already mentioned, the downwelling MMC in the polar tachocline that makes field confinement possible can be regarded as due to a gyroscopically-pumped MMC trying to burrow downward, but held in check by its encounter with the interior field and with the helium settling layer, if present. Now, because the early Sun rotated much faster than today, not only would there have been no helium settling layer but, also, the burrowing tendency would have been much stronger than today, tending to push the bottom of the tachocline downward. This reopens the possibility conjectured in GM98 that there might have been a ventilated “polar pit” or “cauldron” in which most of the Sun’s lithium, though not too much of its beryllium, was burnt during the first gigayear or so of the Sun’s main-sequence evolution.

To take this further we again need to consider the way in which the confinement layer fits into the
global picture. It is arguable that the entire polar downwelling region is depressed relative to its surroundings, forming a wider cauldron, too shallow to burn lithium in today’s Sun but possibly deep enough in the early Sun.

Fig. 5.— Sketch of the magnetic confinement layer and its immediate surroundings at the bottom of the high-latitude tachocline. Close to the pole the interior magnetic field (solid lines) is confined by the downwelling MMC (dashed streamlines). The vertical scale has been greatly exaggerated.

Here we need to distinguish the shape of the ventilated region from the shapes of the stratification surfaces, which latter must remain relatively flat, meaning close to the horizontal. Figure 5 sketches the way in which the confinement layer might fit into its surroundings near the bottom of the polar tachocline. The stratification surfaces are shown dotted. At the periphery of the polar downwelling region, the field lines (solid) emerge from the confinement layer on their way to lower latitudes. They will tend to splay out and slant upward as they exit the downwelling region. The MMC will similarly slant upward, flowing approximately along the field lines (dashed streamlines). This is because the splaying-out increases the magnetic Reynolds number beyond the order-unity values characteristic of the confinement layer. Further out, the field lines must continue to rise through the tachocline until they encounter the convection zone’s overshoot layer, where they are held horizontal by turbulent magnetic flux pumping as suggested in Fig. 1. On the way we must expect turbulent eddy fluxes to become increasingly important, decoupling the MMC’s upwelling streamlines from the time-averaged field lines and leaving the upwelling free to spread over a wide range of latitudes, constrained only by mass conservation and global-scale heat flow.

Such a picture applies equally well to today’s Sun and to the early Sun, the main difference being that the ventilated polar region (unshaded in Fig. 5) is likely to have been pushed deeper in the early Sun with its much faster rotation, stronger burrowing tendency, and global-scale $|B_i|$ values only modestly larger. The ventilated polar region could well have been deeper by many tens of meters, as required to burn lithium.

Within the peripheral lightly-shaded region, into which the MMC does not penetrate, we suggest that ventilation is weak or nonexistent and that shear will be limited by the Ferraro constraint. The darker shading represents the top part of today’s helium settling layer.

As the lightly-shaded region expands upward and outward beyond the immediate surroundings sketched in Fig. 5, through the tachocline toward the overshoot layer, we may surmise that Taylor–Spruit or magnetorotational small-scale MHD instabilities will kick in (e.g., Spruit 2002; Gilman & Cally 2007; Parfrey & Menou 2007, & refs.), breaking the Ferraro constraint and blurring the distinction between the shaded and unshaded regions as turbulent eddy fluxes increase. So a larger-scale picture of the “cauldron”, or “frying pan”, would show its upward-sloping lower boundary becoming increasingly porous and indistinct at greater colatitudes.

The global tachocline model that would be needed to test, and to begin to quantify, the foregoing speculations would have to describe

1. the precise way in which turbulent stresses in the convection zone and tachocline gyroscopically pump the polar downwelling responsible for confining $B_i$;
2. the global-scale distribution of temperature and heat flow that fits in with the MMCs;
3. the turbulent magnetic flux pumping by convective overshoot that we assume confines $B_i$ in extra-polar latitudes;
4. the extent to which the winding-up of the time-averaged toroidal field in extra-polar latitudes (Fig. 1) is limited by turbulent eddy fluxes;
5. the reaction of the overlying turbulent layers to all of the above, especially the deficit in the convection zone’s differential rotation.
governing the torques exerted from above, for instance via feedback on the strength of gyroscopic pumping of the MMC.

Progress on these formidable problems will of course depend on finding suitable ways to model the turbulent processes.

Finally, we note that material transport by burrowing MMCs may not be the only possible explanation for the Sun’s observed lithium and beryllium abundances. For instance early main-sequence mass loss, perhaps followed by lithium production by flares, might have had a role (e.g., Swenson & Faulkner 1992, L. A. Willson, personal communication).


REFERENCES


This 2-column preprint was prepared with the AAS LaTeX macros v5.2.